SHIVALIK SR. SEC. SCHOOL, BHARTHARI ROAD, BEHROR

CLASS XI

TOPIC:- BASIC MATHEMATICS

SUBJECT- Mathematics (Ramesh Suthar Sir)

1. INTEGER EXPONENTS

- **If a is any number and n is a positive integer, then a^n = a.a.a..................a (n times)**
- **$a^0 = 1$; $a \neq 0$**
- **If a is any non – zero numbers and n is a positive integer, then $a^{-n} = \frac{1}{a^n}$**
- **Properties :-**

(1) $a^n.a^m = a^{n+m}$

(2) $(a^n)^m = a^{nm}$

(3) $(ab)^n = a^n b^n$

(4) $\frac{a^n}{a^m} = \begin{cases} a^{n-m} & ; n > m \\ & ; a \neq 0 \\ \frac{1}{a^{m-n}} & ; m > n \end{cases}$

(5) $(\frac{a}{b})^n = \frac{a^n}{b^n}$; $b \neq 0$

(6) $(\frac{a}{b})^{-n} = (\frac{b}{a})^n = \frac{b^n}{a^n}$

(7) $(ab)^{-n} = \frac{1}{(ab)^n}$

(8) $\frac{1}{a^{-n}} = a^n$

(9) $\frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}$

(10) $(a^n b^m)^k = a^{nk} . b^{mk}$

(11) $(\frac{a^n}{b^m})^K = \frac{a^{nk}}{b^{mk}}$

Ques :- Simplify each of the following and write the answere with only positive exponents :-

(a) $(4x^{-4} y^5)^3$

(b) $(-10z^2 y^{-4})^2 . (z^3 y)^{-5}$

(c) $\frac{n^2 m}{7m^{-4} n^3}$

(d) $\frac{5x^{-1}y^{-4}}{(3y^5)^{-2}.\ x^9}$

(e) $(\frac{z^{-5}}{z^{-2}\ x^{-1}})^6$

(f) $(\frac{24a^3 b^{-8}}{6a^{-5}b})^{-2}$

2. RATIONAL EXPONENTS

- **Exponents in the form of $b^{m/n}$; Where both m and n are integers is called rational exponents.**
  - $b^{1/n}$ is called the nth root of b.
  - $a = b^{1/n} \Leftrightarrow a^n = b$

Q.1 Evaluate each of the following : -

(a) $(25)^{\frac{1}{2}}$ (b) $(32)^{\frac{1}{5}}$ (c) $(81)^{\frac{1}{4}}$ (d) $(-8)^{\frac{1}{3}}$ (e) $(16)^{\frac{1}{4}}$ (f) $-16^{\frac{1}{4}}$

Q. 2 Evalute each of the following :-

(a) $(8)^{\frac{2}{3}}$ (b) $(625)^{\frac{3}{4}}$ (c) $(\frac{243}{32})^{\frac{4}{5}}$

3. RADICALS

- **If n is a positive integer that is greater than 1 and a is a real number than. $\sqrt[n]{a} = a^{1/n}$**

  Where the symbol $\sqrt{\ }$ is called redicals :-

- **Properties :-**

  (1) $\sqrt[n]{a^n} = a$ (2) $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

  (3) $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

  Q.1 Write each of the following radicals in exponent form :-

  (a) $\sqrt[4]{16}$ (b) $\sqrt[10]{8x}$ (c) $\sqrt{x^2 + y^2}$

  Q.2 Evaluate each of the following :-

  (a) $\sqrt{16}$ (b) $\sqrt[4]{16}$ (c) $\sqrt[5]{243}$ (d) $\sqrt[4]{1296}$ (e) $\sqrt[3]{-125}$ (f) $\sqrt[4]{256}$

4. PROPORTION

- **When two ratios are equal, then the four quantities compositing then are said to be proportional :-**

  If $\frac{a}{b} = \frac{c}{d} \implies a : b = c : d$

  $\implies a : b :: c : d$

- **Product of external terms = Product of internal terms**
- **a x d = b x c**
- If $\frac{a}{b} = \frac{c}{d} \implies \frac{b}{a} = \frac{d}{c}$ (Reciprocal Laws)
- If $\frac{a}{b} = \frac{c}{d} \implies \frac{a}{c} = \frac{b}{d}$ (Alternate Laws)
- If $\frac{a}{b} = \frac{c}{d} \implies \frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)
- If $\frac{a}{b} = \frac{c}{d} \implies \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)
- If $\frac{a}{b} = \frac{c}{d} \implies \frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo & Dividendo)

5. LOGARITHM

❖ **Defination :-** 2f a, n, x are three real number such that a > 0, x > 0 and nϵR

$\therefore a^n = x \implies \log_a x = n$ (Logarithm form)

$\therefore \log_a x = n \implies a^n = x$ (Exponent form)

Composite form :- $a^n = x \Leftrightarrow \log_a x = n$

- **Examples :-**

(i) $10^{-3} = 0.001 \Longrightarrow \log_{10} 0.001 = --3$

(ii) $\log_4 256 = 4 \Longrightarrow 4^4 = 256$

Note :- $a^n = x \Longrightarrow \log_a x = n$, Base of logarithm = a, Argument = x

- **Properties of logarithm :-**

If M, N > 0 and a > 0, b > 0

(i) Addition Law :-

$\log_a MN = \log_a M + \log_a N$

Proof :- Let $\log_a M = x \Longrightarrow M = a^x$ .... (1)

and $\log_a N = y \Longrightarrow N = a^y$ .... (2)

Multiply by equations (1) and (2), we get

⇨ $MN = a^x.a^y$

⇨ $MN = a^{x+y}$ $\{\therefore a^m.\ a^n = a^{m+n}\}$

Convert into logarithm,

⇨ $\log_a (MN) = xy$

Value put from equation (1) & (2), We get

⇨ $\log_a MN = \log_a M + \log_a N$

(ii) Difference Law :-

$\log_a (\frac{M}{N}) = \log_a M + \log_a N$

Proof :- Let $\log_a M = x \Longrightarrow M = a^x$ .... (1)

and $\log_a N = y \Longrightarrow N = a^y$ .... (2)

Multiply by equations (1) and (2), we get

⇨ $\frac{M}{N} = \frac{a^x}{a^y}$

⇨ $\frac{M}{N} = a^{x-y}$ $\{\therefore a^m \div a^n = a^{m-n} ; m > n\}$

Convert into logarithm,

⇨ $\log_a (\frac{M}{N}) = x - y$

Value put from equation (1) & (2), We get

⇨ $\log_a (\frac{M}{N}) = \log_a M - \log_a N$

(iii) Multiplication Law :-

$\log_a M^N = N.\log_a M$

Proof :- Let $\log_a M = x \Longrightarrow M = a^x$

⇨ $M^N = (a^x)^N$

⇨ $M^N = a^{Nx}$

Convert into logarithm,

⇨ $\log_a M^N = N . x$

put the value of x ,we get

$\Longrightarrow \log_a M^N = N.\log_a M$

(iv) $a^{\log_a M} = M$

Proof :- Let $\log_a M = x$

$\Longrightarrow a^x = M$

put the value of x ,we get

$\Longrightarrow a^{\log_a M} = M$

(v) <u>Base convert formula</u> :-

$$\log_N M = \frac{\log_a M}{\log_a N} = \frac{\log_e M}{\log_e N} = \frac{\log_{10} M}{\log_{10} N} = \ldots\ldots\ldots\ldots\ldots\ldots$$

OR

$\log_N M \times \log_a N = \log_a M$

Proof :- Let $\log_N M = x \Longrightarrow N^x = M$ .... (1)

and $\log_a N = y \Longrightarrow a^y = N$ .... (2)

Put the value of N from equation (2) to equation (1)

$\Longrightarrow (a^y)^x = M$

$\Longrightarrow a^{xy} = M$

Convert into logarithm

$\Longrightarrow \log_a M = xy$

Put the value of x and y in from equation (1) and (2)

$\Longrightarrow \log_a M = \log_N M \times \log_a N$

$\Longrightarrow \log_N M = \frac{\log_a M}{\log_a N}$

(vi) $\log_b a \times \log_a b = 1 \Longrightarrow \log_b a = \frac{1}{\log_a b}$

Proof :- Let $\log_b a = x \Longrightarrow b^x = a$ .... (1)

and $\log_a b = y \Longrightarrow a^y = b$ .... (2)

Put the value of b from equation (2) to equation (1)

$\Longrightarrow (a^y)^x = a$

$\Longrightarrow a^{xy} = a^1$ $\{\therefore a^x = a^y \Longleftrightarrow x = y\}$

$\Rightarrow xy = 1$

Convert into logarithm

$\Rightarrow \log_a M = xy$

Put the value of x and y in from equation (1) and (2)

$\Rightarrow$ **$\log_b a \times \log_a b = 1$**

(vii) $\log_a a = 1$; $a \neq 1$

Proof :- We know that

$\therefore a^1 = a$

$\Rightarrow \log_a a = 1$

$\therefore \log_e e = 1$

$\therefore \log_{10} 10 = 1$

$\therefore \log_2 2 = 1$

(viii) $\log_a 1 = 0$; $a \neq 1$

Proof :-

$\therefore a^0 = 1$

Convert into logarithm

$\Rightarrow \log_a 1 = 0$

$\therefore \log_2 1 = 0$

$\therefore \log_{10} 1 = 0$

$\therefore \log_e 1 = 0$

(ix) $\log_a 0 = -\infty$; $a > 1, a \neq 0$

Proof :- $\therefore a^{-\infty} = 0$

$\Rightarrow \log_a 0 = -\infty$

(x) $\log_a 0 = +\infty$; $a < 1, a \neq 0$

Proof :-

$\therefore a < 1$

Let $a = \frac{1}{b}$ then $b > 1$

$\therefore a^{-\infty} = 0$

$\Rightarrow \left(\frac{1}{b}\right)^{-\infty}$

$\Rightarrow \frac{1}{b^{-\infty}} = 0$

$a^{-\infty} = 0$

$a^{\infty} = \infty$

$a^0 = 1$

- **Kinds of logarithm** :-

(i) Common logarithm :-

Logarithm which base has 10.

eg :- $\log_{10} x$, $\log_{10} e$, ...

(ii) Natural logarithm :-

Logarithm which base has e.

eg :- $\log_e 2$, $\log_e x$, ...

$\log_e x = \ln x$

- **Relation between common & Naturral logarithm**

Using Base – convert formula ,

$\therefore \log_e x = \frac{\log_{10} x}{\log_{10} e}$

$\Longrightarrow \log_e x = (\log_e x) \times (\log_{10} x)$

$\Longrightarrow \ln x = (\log_e x) \times (\log_{10} x)$

$\Longrightarrow \ln x = 2.303 \log_{10} x$

-: Excercise :-

Write the following in logarithm form :-

1. $2^6 = 64$ 2. $10^4 = 10000$ 3. $2^{10} = 1024$

4. $5^{-2} = \frac{1}{25}$ 5. $10^{-3} = 0.001$ 6. $(4)^{3/2} = 8$

Write the following in the exponent form :-

7. $\log_5 25 = 2$ 8. $\log_3 729 = 6$ 9. $\log_{10}(0.001) = -3$

10. $\log_{10}(0.1) = -1$ 11. $\log_3(1/27) = -3$ 12. $\log_{\sqrt{2}} 4 = 4$

13. If $\log_{81} x = \frac{3}{2}$, then find the value of x.

14. If $\log_{125} P = \frac{1}{6}$, then find the value of P.

15. If $\log_4 M = 1.5$, then find the value of m.

16. Prove that :- log630 = log2 + 2log3 + log5 + log7

17. Prove that :- log10 + log100 + log1000 + log10000 = 10

18. Prove that :- $\log(\frac{9}{14}) + \log(\frac{35}{25}) - \log(\frac{15}{16}) = 0$

19. If log2 = 0.3010, log3 = 0.4771, log7 = 0.8451 and log11 = 1.0414, then find the value of following :-

(i) log36 (ii) $\log(\frac{42}{11})$ (iii) $\log(\frac{11}{7})^5$ (iv) log70 (v) $\log\frac{121}{120}$ (vi) $\log 5^{1/3}$

20. Find the value of *x* from following equations :-

$\log_x 4 + \log_x 16 + \log_x 64 = 12$

21. Solve the equation :- $\log(x + 1) - \log(x - 1) = 1$

22. Find the value :- $3^{2-\log_3 4}$

23. Find :- (i) log2 + 1 (ii) $\log_5 3 . \log_3 4 . \log_2 5$

24. If log2 = 0.3010, then find the value of log200.

25. Find the value of log6 + 2log5 + log4 – log3 – log2

6. POLYNOMIAL

- **Remainder Theorm** :- Let P(x) be any polynomial of degree greater than or equal to one
  and Let a be any real number . If P(x) is divided by the linear polynomial (x-a) , **then the remainder is P(a)** .
- **Factor Theorm :-** Let P(x) is a polynomial of degree n $\geq$ 1 and a is any real number, then :-
  (i) (x - a) is a factor of P (x), if P(a) = 0, and
  (ii) P(a) = 0, if (x – a) is a factor of P(x).
  i.e. (x –a) is a factor of P (x) $\Leftrightarrow$ P(a) = 0
- **Algebraic Identities :-**

1. $(a + b)^2 = a^2 + 2ab + b^2 = (a – b)^2 + 4ab$
2. $(a + b)^2 = a^2 - 2ab + b^2 = (a + b)^2 - 4ab$
3. $a^2 – b^2 = (a + b) (a – b)$
4. $(a + b)^3 = a^3 + b^3 + 3ab (a + b)$
5. $(a - b)^3 = a^3 - b^3 - 3ab (a - b)$
6. $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$
7. $a^3 - b^3 = (a - b) (a^2 + ab + b^2)$
8. (i) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$
   (ii) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2abc (\frac{1}{a} + \frac{1}{b} + \frac{1}{c})$
9. $a^2 + b^2 + c^2 – ab – bc – ac = \frac{1}{2} [(a – b)^2 + (b – c)^2 + (c – a)^2]$
10. $a^3 + b^3 + c^3 – 3abc = (a + b + c) (a^2 + b^2 + c^2 – ab – bc – ca)$
11. If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$
12. $a^4 + a^2 + 1 = (a^2 + a + 1) (a^2 – a + 1)$

-: Exercise :-

1. $6x^7 + 3x^4 – 9x^3$	2. $a^3b^8 – 7a^{10}b^4 + 2a^5b^2$	3. $2x + (x^2 + 1)^3 – 16 (x^2 + 1)^5$
4. $x^2 + (2 - 6x) + 4x (4 - 12x)$		5. $x^2 - 2x – 8$
6. $z^2 – 10z + 21$	7. $y^2 + 16y + 60$	8. $5y^2 + 14y – 3$
9. $6t^2 – 19t – 7$	10. $4z^2 + 19z + 12$	11. $z^2 + 14z + 49$
12. $4w^2 – 25$	13. $81x^2 – 36x + 4$	14. $12x^2 – 7x + 1$
15. $2x^2 – 7x + 3$	16. $6x^2 + 5x – 6$	17. $3x^2 – x – 4$
18. $y^2 – 5y + 6$	19. $6x^2 + 13x + 5$	20. $x^3 – 2x^2 - x + 2$
21. $x^3 – 3x^2 - 9x – 5$	22. $x^3 + 13x^2 + 32x + 20$	23. $2y^3 + y^2 – 2y – 1$
24. $x^3 – 23x^2 + 142x – 120$		

7. QUADRATIC EQUATION

- **An equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, $a,b,c \in R$ is called a quardraticequation with real coefficients.**
- A quadratic equation has two degree equation.
- The quantity $D = b^2 - 4ac$ is known as the discriminant of the quadratic equations.
- Roots of quadratic equation are given by :-

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where :- a = Coefficient of x^2

b = Coefficient of x

c = constant

- **Nature of roots :- $D = b^2 - 4ac$**
  1. The roots are real and distinct iff $D > 0$
  2. The roots are real and equal iff $D = 0$
  3. The roots are complex with non – zero imaginary part iff $D < 0$
  4. The roots are rational iff a, b, c are rational and D is a perfect square.
  5. The roots are irrational iff a, b, c are rational and D is not a perfect square.
  6. Quadratic equation has reciprocal root if C = O
- **Relation between coefficient and Roots of quadaric equations :-**

  Let α and β are two roots of a quadratic equation

  $ax^2 + bx + c = 0$,then

  $\therefore \alpha + \beta = -\frac{\text{Coeff .of } x}{\text{Coeff .of } x^2}$ $\therefore \alpha\beta = \frac{\text{Constant}}{\text{Coeff .of } x^2}$

  $\Longrightarrow \alpha + \beta = -\frac{b}{a}$ $\Longrightarrow \alpha\beta = \frac{c}{a}$

- **Symmetric function :- Let α and β are two roots of a quadratic equation $ax^2 + bx + c = 0$, then $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$ so α and β are called a symmetric function. If function is invariant when α and β substitude mutually.**
- Some useful results :-
  1. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$
  2. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$
  3. $\alpha^4 + \beta^4 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$
  4. $\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$
  5. $\alpha^2 - \beta^2 = (\alpha + \beta)\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$

6. $\alpha^3 - \beta^3 = [(\alpha + \beta)^2 - \alpha\beta](\alpha + \beta)$

7. $\alpha^4 - \beta^4 = (\alpha + \beta).\sqrt{(\alpha + \beta)^2 - 4\alpha\beta}\,.\,[(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)]$

-: Exercise :-

1. If α and β are roots of equation $2x^2 - 5x + 7 = 0$, then find the equation with roots $(2\alpha + 3\beta)$ and $(3\alpha + 2\beta)$.
2. If $a \neq b$, then find the nature of quadratic equation $2(a^2 + b^2)x^2 + 2(a + b)x + 1 = 0$.
3. If α and β are roots of equation $px^2 + qx + r = 0$, then find the value of $\alpha^3\beta + \beta^3\alpha$.
4. If the product of roots of equation $mx^2 + 6x + (2m - 1) = 0$ is – 1, then find the value of m.
5. The roots of equation $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ are common root, then find value of k .
6. For equation $3x^2 + px + 3 = 0$, $p > 0$, if one root is square of other root, then find the value of p .
7. In quadratic equation $x^2 + px + q = 0$ roots are $\tan 30^0$ and $\tan 15^0$ respectively, then find the value of $(2 + q - p)$.
8. Let α, β are roots of equation $x^2 - px + r = 0$ and $\alpha/2$, 2β are roots of equation $x^2 - qx + r = 0$ then find the value of r.
9. If equation $x^2 + ax + 12 = 0$ has a root 4, such that equation $x^2 + ax + b = 0$ has equal roots, then find b.
10. If a, b, c are three sides of a triangle, such that equation $x^2 - 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ have real roots, find the value of λ.

8. CUBIC EQUATION

- **An equation $ax^3 + bx^2 + cx + d = 0$; $a \neq 0$, a, b, c, d ϵ R is known as cubic equation with real coefficients.**
- The degree of a cubic equation is 3.
- If α, β, γ are roots of cubic equation, then relation between roots and coefficient are given below :-

$$\alpha + \beta + \gamma = -\frac{\text{Coeff .of } x^2}{\text{Coeff .of } x^3} = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{\text{Coeff .of } x}{\text{Coeff .of } x^3} = -\frac{c}{a}$$

$$\alpha\beta\gamma = -\frac{\text{Constant}}{\text{Coeff .of } x^3} = -\frac{d}{a}$$

- Equation of roots α, β and γ is :-

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma)x - \alpha\beta\gamma = 0$$