# SHIVALIK SR. SEC. SCHOOL, BHARTHARI ROAD, BEHROR

## CLASS XI (LESSON- I)

## Units, Dimensions and Measurement

## SUBJECT- Physics (Ajay Kumar Gupta Sir)

- **Physical Quantities:-**

All the quantities in the term of which laws of physics are described, and whose measurement is necessary are called physical quantities. For example length, mass, time etc. Physical quantities are of two types :

- **Fundamental and Derived Quantities:-**

**(1) Fundamental quantites:-** Those physical quantities which are independent to each other are called fundamental quantities and their units are called fundamental units. eg: mass, time, length etc.

**(2) Derived quantites:-** Those physical quantities which are derived from fundamental quantities are called derived quantities and their units are called derived units. eg: velocity, force, work etc.

**Units :-** A definite amount of a physical quantity ithe nus taken as its standard unit. The unit should be easily reproducible, internationally accepted.

The unit of physical quantity is inversely proportional to numeric value i.e. $\mathbf{u \propto 1/n}$ where u is unit of physical quantity and n is the numeric value.

**System of units:** A complete set of units, both fundamental and derived for all kinds of physical quantities is called system of units. The common systems are given below:

**(1) CGS system:** The system is also called Gaussian system of units. In this *length, mass* and *time* have been chosen as the fundamental quantites and corresponding fundamental units *are centimetre (cm), gram (gm) and second (s)* respectively.

(2) **MKS system:** This system is also called Giorgi system of units. In this length mass and time have been taken as fundamental quantities, and the corresponding fundamental units are *metre, kilogram and second.*

(3) **FPS system:** In this system *foot, pound and second* are used respectively for measurements of length, mass and time.

(4) **S. I. system:** It is known as International system of units, and is extended system of units applied to whole physics. There are seven fundamental quantities in this system. These quantities and their units are given in the following table:

**Unit and symbol of quantities**

| Quantity | Unit | Symbol |
|---|---|---|
| Length | Metre | m |
| Mass | Kilogram | kg |
| Time | Second | s |
| Electric current | Ampere | A |
| Temperature | Kelvin | K |
| Amount of Substance | Mole | mol |
| Luminous Intensity | Candela | cd |

Besides the above seven fundamental units two supplementary units are also defined- Radian (rad) for plane angle and Steradian (sr) for solid angle.

**Note:-** Apart from fundamental and derived units we also use practical units very frequently. These may be fundamental or derived units e.g. light year is a practical unit (fundamental) of distance while horse power is a practical unit (derived ) of power.

Pracitcal units may or may not belong to a system but can be expressed in any system of units.

e.g. 1 mile = 1.6 km = $1.6 \times 10^3$m.

- **S.I. Prefixes:**

In Physics we deal from very small (micro) to very large (macro) magnitudes as one side we talk about the atom while on the other side of universe. E.g., the mass of an electron is $9.1 \times 10^{-31}$ kg while that of the sun is $2 \times 10^{30}$ kg. To express such large or small magnitudes we use the following prefixes:

**Prefxes and symbol**

| Power of 10 | Prefix | Symbol |
|---|---|---|
| $10^{18}$ | Exa | E |
| $10^{15}$ | Peta | P |
| $10^{12}$ | tera | T |
| $10^{9}$ | giga | G |
| $10^{6}$ | mega | M |
| $10^{3}$ | Kilo | k |
| $10^{2}$ | Hecto | h |
| $10^{1}$ | deca | da |

| | | |
|---|---|---|
| $10^{-1}$ | deci | d |
| $10^{-2}$ | centi | c |
| $10^{-3}$ | milli | m |
| $10^{-6}$ | micro | µ |
| $10^{-9}$ | nano | n |
| $10^{-12}$ | pico | p |
| $10^{-15}$ | femto | f |
| $10^{-18}$ | atto | a |

## ➢ Standards of Length, Mass and Time

(1) **Length:** Standard metre is defined in terms of wavelength of light and is called atomic standard of length.

The metre is the distance countaining 1650763.73 wavelength in vacuum of the radiation corresponding to orange -red light emitted by an atom of Krypton-86.

Now a days metre is defined as length of the path travelled by light in vacuum in 1/299, 792, 458 part of a second.

(2) **Mass:** The mass of a cylinder made of platinum-iridium alloy kept at International bureau of Weights and Measures is defined as 1 kg.

On atomic scale, 1 kilogram is equivalent to the mass of 5.0188 x $10^{25}$ atoms of C-12 (an isotope of carbon).

(3) **Time:** 1 second is defined as the time interval of 9192631770 vibrations of radiation in Cs- 133 atom. This radiation corresponds to the transition between two hyperfine level of the ground state of Cs-133.

## ➢ Pratical Units:

(1) **Length**

(i) 1 fermi = 1 fm = $10^{-15}$ m

(ii) 1 X-ray unit = 1 XU = $10^{-13}$ m

(iii) 1 angstrom = 1 $A^o$ = $10^{-10}$ m = $10^{-8}$ cm = $10^{-7}$ mm = 0.1 µm

(iv) 1 micron = 1 µm = $10^{-6}$ m

(v) 1 astronomical unit = 1 AU = 1.49 x $10^{11}$ m

= 1.5 x $10^{11}$ m = 1.5 x $10^{8}$ km

(vi) 1 light year = 1 ly = 9.46 x $10^{15}$ m

(vii) 1 Parsec = 1pc = 3.26 light year = 3.08 x $10^{16}$m

(2) **Mass**

(i) Chandra Shekhar unit = 1 CSU = 1.4 times the mass of sun = $2.8 \times 10^{30}$ kg

(ii) 1 tonne = 1 Metric ton = 1000kg

(iii) Quintal = 1 Quintal = 100kg

(iv) Atomic mass unit (amu) : amu = $1.67 \times 10^{-27}$ kg

Mass of proton or neutron is of the order of 1amu

(3) **Time**

(i) Year : It is the time taken by the Earth to complet 1 revolution around the Sun in its orbit.

(ii) Lunar month : It is the time taken by the Moon to complete 1 revolution around the Earth in its orbit.

1 LM = 27.3 days

(iii) Solar day: It is the time taken by Earth to complete one rotation about its axis with respect to Sun. Since this time varies from day to day,average solar day is calculated by taking average of the duration of all the days in a year and this is called Average solar day.

1 solar year = 365.25 average solar day

Or average solar day = $\frac{1}{365.25}$ the part of solar year

(iv) Sediral day : It is the time taken by earth to complete one rotation about its axis with respect to a distant star.

1 Solar year = 366.25 Sedrial day

= 365.25 average solar day

Thus 1 Sedrial day is less than 1 solar day.

(v) Shake: It is an obsolete and practical unit of time.

1 Shake = $10^{-8}$ sec.

➢ **Dimensions:**

When a derived quantity is expressed in terms of fundamental quantities it is written as a product of different powers of the fundamental quantities. The powers to which fundamental quantities must be raised in order to express the given physical quantity are called its dimensions.

To make it more clear, consider the physical quantity force

Force = mass x acceleration = $\frac{mass \times velocity}{time}$

$= \frac{mass \times length/time}{time}$

= mass x length x $(time)^{-2}$ ..................................(1)

Thus, the dimensions of force are 1 in mass, 1 in length and -2 in time.

Here the physical quantity that is expressd in terms of the basic quantities is enclosed in square brackets to indicate that the equation is among the dimensions and not among the magnitudes.

Thus equation (1) can be written as [force] = $[MLT^{-2}]$

Such an expression for a physical quantity in terms of the fundamental quantities is called the dimensional equation. If we consider only the R.H.S. of the equation, the expression is termed as dimensional formula

Thus, dimensional formula for force is. $[MLT^{-2}]$

- **Quantities Having same Dimensions**

| **Dimension** | Quantity |
|---|---|
| $[M^0 L^0 T^{-1}]$ | Frequency, angular frequency, angular velocity, velocity gradient and decay constant |
| $[M^1 L^2 T^{-2}]$ | Work, internal energy, potential energy, kinetic energy, torque, moment of force |
| $[M^1 L^{-1} T^{-2}]$ | Pressure, stress, young's modulus, bulk modulus, modulus of rigidity, energy density |
| $[M^1 L^1 T^{-1}]$ | Momentum, impulse |
| $[M^0 L^1 T^{-2}]$ | Acceleration due to gravity, gravitional field intensity |
| $[M^1 L^1 T^{-2}]$ | Thrust, force, weight, energy gradient |
| $[M^1 L^2 T^{-1}]$ | Angular momentum and Planck's constant |
| $[M^1 L^0 T^{-2}]$ | Surface tension, surface energy (energy per unit area) |
| $[M^0 L^0 T^0]$ | Strain, refractive index, relative density, angle, solid angle, distance gradient, relative permittivity (dielectric constant), relative permeability etc. |
| $[M^1 L^2 T^{-2} \Theta^{-1}]$ | Thermal capacity, gas constant, Boltzmann constant and entropy |
| $[M^0 L^0 T^1]$ | $\sqrt{l/g}, \sqrt{m/k}, \sqrt{R/g},$ where l = length<br>g = acceleration due to gravity, m = mass k = spring constant, R = Radius of earth |
| $[M^0 L^0 T^1 A^0]$ | L/R, $\sqrt{LC}$ ,RC where L = inductance, R = resistance, C = capacitance |
| $[M^1 L^2 T^{-2} A^0]$ | $I^2$ Rt, $\frac{V^2}{R}t$, VIt, qV, $LI^2$, $\frac{q^2}{C}$, $CV^2$ where I = current, t = time, q = charge,<br>L = inductance, C = capacitance, R = resistance |

## ➢ **Important Dimensions of Complete Physics**

### Heat

| Quantity | Unit | Dimension |
|---|---|---|
| Temperature (T) | Kelvin | $[M^0L^0T^0\,\theta^1]$ |
| Heat (Q) | Joule | $[M^1L^2T^{-2}]$ |
| Specific Heat (c) | Joule/( kgxK) | $[M^0L^2T^{-2}\,\theta^{-1}]$ |
| Thermal capacity | Joule/K | $[M^1L^2T^{-2}\,\theta^{-1}]$ |
| Latent heat (L) | Joule/kg | $[M^0L^2T^{-2}]$ |
| Gas constant (R) | Joule/(molxK) | $[M^1L^2T^{-2}\,\theta^{-1}]$ |
| Boltzmann constant (k) | Joule/K | $[M^1L^2T^{-2}\,\theta^{-1}]$ |
| Coefficient of thermal conductivity (K) | Joule/m-s-K | $[M^1L^1T^{-3}\,\theta^{-1}]$ |
| Stefan's constant (σ) | Watt/ $m^2$ $K^4$ | $[M^1L^0T^{-3}\,\theta^{-4}]$ |
| Wein's constant (b) | Metre-K | $[M^0L^1T^0\,\theta^1]$ |
| Planck's constant (h) | Joule-s | $[M^1L^2T^{-1}]$ |
| Coefficient of Linear Expension (α) | $Kelvin^{-1}$ | $[M^0L^0T^0\,\theta^{-1}]$ |
| Mechanical equivalent of Heat (J) | Joule/ Calorie | $[M^0L^0T^0]$ |

### Electricity

| Quantity | Unit | Dimension |
|---|---|---|
| Electric charge (q) | Coulomb | $[M^0\,L^0\,T^1\,A^1]$ |
| Electric current (I) | Ampere | $[M^0\,L^0\,T^0\,A^1]$ |
| Capacitance (C) | Coulomb/ volt or Farad | $[M^{-1}\,L^{-2}\,T^4\,A^2]$ |
| Electric potential (V) | Joule/ coulomb | $[M^1\,L^2\,T^{-3}\,A^{-1}]$ |
| Permittivity of free space (ε) | $\frac{\text{Coulomb}^2}{Newton-\text{metre}^2}$ | $[M^{-1}\,L^{-3}\,T^4\,A^2]$ |
| Dielectric constant (K) | Unitless | $[M^0\,L^0\,T^0]$ |
| Resistance (R) | Volt/ Ampere or ohm | $[M^1\,L^2\,T^{-3}\,A^{-2}]$ |
| Resistivity or Specific resistance (ρ) | Ohm- metre | $[M^1\,L^3\,T^{-3}\,A^{-2}]$ |
| Coefficient of Self inductance (L) | $\frac{\text{volt}-\text{second}}{ampere}$ or henry or ohm-second | $[M^1\,L^2\,T^{-2}\,A^{-2}]$ |
| Magnetic fulx (ϕ) | Volt-second or weber | $[M^1\,L^2\,T^{-2}\,A^{-1}]$ |

| | | |
|---|---|---|
| Magnetic induction (B) | $\frac{\text{newton}}{ampere - metre}$ <br> $\frac{\text{Joule}}{ampere - metre^2}$ <br> $\frac{\text{volt} - \text{second}}{metre^2}$ or Tesla | $[M^1 T^{-2} A^{-1}]$ |
| Magnetic Intensity (H) | Ampere/metre | $[L^{-1} A^1]$ |
| Magnetic Dipole Moment (M) | Ampere-metre$^2$ | $[M^0 L^2 T^0 A^1]$ |
| Permeability of Free Space (μ) | $\frac{\text{Newton}}{ampere^2}$ <br> Or $\frac{\text{joule}}{ampere^2 - \text{metre}}$ <br> Or $\frac{\text{Volt} - \text{second}}{\text{ampere} - \text{metre}}$ <br> Or $\frac{\text{Ohm} - \text{second}}{\text{metre}}$ <br> Or $\frac{\text{henry}}{\text{metre}}$ | $[M^1 L^1 T^{-2} A^{-2}]$ |
| Surface charge density | Coulomb metre$^{-2}$ | $[M^0 L^{-2} T^1 A^1]$ |
| Electric dipole moment (p) | Coulomb- metre | $[M^0 L^1 T^1 A^1]$ |
| Conductance (G) (1/R) | Ohm$^{-1}$ | $[M^{-1} L^{-2} T^3 A^2]$ |
| Conductivity (σ) (1/ρ) | Ohm$^{-1}$metre$^{-1}$ | $[M^{-1} L^{-3} T^3 A^2]$ |
| Current density (J) | Ampere/m$^2$ | $[M^0 L^{-2} T^0 A^1]$ |
| Intensity of electric field (E) | Volt/ metre, Newton/ coulomb | $[M^1 L^1 T^{-3} A^{-1}]$ |
| Rydberg constant (R) | m$^{-1}$ | $[M^0 L^{-1} T^0]$ |

- **Application of Dimensional Analysis**

(1) **To find the unit of a physical quantity in a given system of units:**

To write the definition or formula for the physical quantity we find its dimensions. Now in the dimensional formula replacing M, L and T by the fundamental units of the required system we get the unit of physical quantity. However, sometimes to this unit we further assign a specific name,

e.g. Work = force x Displacement

So $[W] = [M^1 L^1 T^{-2}] \times [L] = [M^1 L^2 T^{-2}]$

So its unit in C.G.S. system will be ***gcm²/s²*** which is called ***erg*** while in M.K.S. system will be ***kg-m²/s²*** which is called ***joule.***

(2) **To find dimensions of physical constant or coefficients:** As dimensions of a physical quantity are unique, we write any formula or equation incorportaing the given constant and then by substituting the dimensional formulae of all other quantities, we can find the dimensions of the required constant or coefficient.

(i) Gravitational constant: According to Newton's law of gravitation

$$F = G\,\frac{m_1 m_2}{r^2} \text{ or } G = \frac{Fr^2}{m_1 m_2}$$

**Substituting the dimensions of all physical quantitites**

$$G = \frac{[MLT^{-2}]\,[L^2]}{[M][M]} = [M^{-1}\,L^{3}T^{-2}]$$

(ii) Planck constant: According to planck $E = h\upsilon$ or $h = \frac{E}{\upsilon}$

**Substituting the dimensions of all physical quantitites**

$$[h] = \frac{[ML^2\,T^{-2}]}{[T^{-1}]} = [M^{1}\,L^{2}T^{-1}]$$

(iii) Coefficient of viscosity: According to Poiseuille's formula

$$\frac{dv}{dt} = \frac{\Pi p r^4}{8\eta l} \text{ or } \eta = \frac{\Pi p r^4}{8l\,(dv/dt)}$$

**Substituting the dimensions of all physical quantitites**

$$[\eta] = \frac{[ML^{-1}\,T^{-2}][L^4]}{[L][L^3/T]} = [M\,L^{-1}T^{-1}]$$

(3) **To convert a physical quantity from one system to the other:** The measure of physical quantity is nu = constant

If a physical quantity X has dimensional formula ($M^a\,L^b\,T^c$) and if (derived) units of that physical quantity in two systems are $\left[M_1^a\,L_1^b\,T_1^c\right]$ and $\left[M_2^a\,L_2^b\,T_2^c\right]$ respectively and $n_1$ and $n_2$ be the numerical values in the two systems respectively, then $n_1\,[u_1] = n_2\,[u_2]$

$$\Rightarrow \quad n_1\left[M_1^a\,L_1^b\,T_1^c\right] = n_2\left[M_2^a\,L_2^b\,T_2^c\right]$$

$$\Rightarrow \quad n_2 = n_1\,\left[\frac{M_1}{M_2}\right]^a\left[\frac{L_1}{L_2}\right]^b\left[\frac{T_1}{T_2}\right]^c$$

Where M, L and T are fundamental units of mass, length and time in the first (known) system and M, L and T are fundamental units of mass, length and time in the second (unknown) system. Thus knowing the values of fundamental units in two systems and numerical value in one system, the numerical value in other system may be evaluated.

**Example: (i)** conversion of Newton into Dyne.

The Newton is the S.I. unit of force and has dimensional formula ($MLT^{-2}$)

So 1 N = 1 kg-m/s$^2$

By using $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$= 1 \left[\frac{\text{Kg}}{\text{gm}}\right]^1 \left[\frac{\text{m}}{\text{cm}}\right]^1 \left[\frac{\text{sec}}{\text{sec}}\right]^{-2}$

$= 1 \left[\frac{10^3\text{gm}}{\text{gm}}\right]^1 \left[\frac{10^2\text{cm}}{\text{cm}}\right]^1 \left[\frac{\text{sec}}{\text{sec}}\right]^{-2} = 10^5$

∴ 1 N = $10^5$ Dyne

**(ii)** Conversion of gravitional constant (G) from C.G.S. to M.K.S system

The value of G in (C.G.S) system is 6.67 x $10^{-8}$ C.G.S. units while its dimensional formula is $(M^{-1} L^3 T^{-2})$

So G = 6.67 x $10^{-8}$ cm$^3$/g s$^2$

By using $n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$

$= 6.67 \times 10^{-8} \left[\frac{\text{gm}}{\text{kg}}\right]^{-1} \left[\frac{\text{cm}}{\text{m}}\right]^3 \left[\frac{\text{sec}}{\text{sec}}\right]^{-2}$

$= 6.67 \times 10^{-8} \left[\frac{\text{gm}}{10^3\text{gm}}\right]^{-1} \left[\frac{\text{cm}}{10^2\text{cm}}\right]^3 \left[\frac{\text{sec}}{\text{sec}}\right]^{-2}$

= 6.67 x $10^{-11}$

∴ G = 6.67 x $10^{-11}$ M.K.S. unit

(4) **To check the dimensional correctness of a given physical relation:** This is based on the principle of homogeneity. According to this principle, the dimensions of each term on both sides of an equation must be the same.

If $X = A \pm (BC)^2 \pm \sqrt{DEF}$

Then, according to principle of homogeneity

$[X] = [A] = [(BC)^2] = [\sqrt{DEF}]$

If the dimensions of each term on both sides are same, the equation is dimensionally correct, otherwise not. A dimensionally correct equation may or may not be physically correct.

**Example: (i)** $F = mv^2 / r^2$

By substituting dimension of the physical quantities in the relation,

$[MLT^{-2}] = [M] [LT^{-1}]^2 / [L]^2$

i.e. $[MLT^{-2}] = [MT^{-2}]$

As in the above equation dimensions of both sides are not same; this formula is not correct dimensionally, so can never be physically.

(5) **As a research tool to derive new relations:** If one knows the dependency of a physical quantity on other quantities and if the dependency is of the product type, then using the method of dimensional analysis, relation between the quantities can be derived.

**Example:** (i) Time period of a simple pendulum.

Let time period of a simple pendulum is a function of mass of the bob (m), effective length (l), acceleration due to gravity (g) then, assuming the function to be product of power of function of m, l and g

i.e. $T = K m^x l^y g^z$ : where K = dimensionless constant

If the above relation is dimensionally correct then by substituting the dimensions of quantities-

$[T] = [M] [L] [LT^{-2}]$ or $[T] = [ML^2 T^{-2}]$

Equating the exponents of similar quantities x = 0, y = ½ and z = -1/2

So the required physical relation becomes $T = K\sqrt{\frac{l}{g}}$

The value of dimensionless constant is found (2Π) through experiments so $T = 2\Pi\sqrt{\frac{l}{g}}$

- **Limitations of Dimensional Analysis:**

Although dimensional analysis is very useful it cannot lead us too far as:

(1) If dimensions are given, physical quantity may not be unique as many physical quantities have same dimensions. For example, if the dimensional formula of a physical quantity is $[ML^2T^{-2}]$ it may be work or energy or torque.

(2) Numerical constant having no dimensions [K] such as (1/2) , 1 or 2Π etc. cannot be deduced by the methods of dimensions.

(3) The method of dimensions can not be used to derive relations other than product of power functions. For example:

$s = ut + (1/2)at^2$ or $y = a \sin\omega t$

Cannot be derived by using this theory. However, the dimensional correctness of these can be checked.

(4) If a quantity depends on more than three factors, having dimensions, the formula cannot be derived. This is because on equating the powers of M,L and T on other side of the dimensional equation, we can obtain three equations, from which only three unknown dimensions can be calculated.

(5) We cannot derive the formulae containing trignometrical functions, exponential functions, logarithm functions etc. which have no dimensions.

(6) It gives no information whether a physical quantity is a scalar or a vector.

- **Significant Figures:**

Significant figures in the measured value of a physical quantity tells the number of digits in which we have confidence. Larger the number of significant figure obtained in a measurement, greater is the accuracy of the measurement. The reverse is also true.

The following rules are observed in counting the number of signicicant figures in a given measured quantity.

**(1) All non- zero digits are significant.**

Example: 42.3 have three significant figures.

423.4 have four significant figures.

24.123 have five significant figures.

**(2) A zero becomes significant fugure if it appears between two non- zero digits.**

Example: 5.03 have three significant figures.

5.604 have four significant figures.

4.004 have four significant figures.

**(3) Leading zeros or the zeros placed to the left of the number are never significant.**

Example: 0.543 have three significant figures.

0.045 have two significant figures.

0.006 have one significant figure.

**(4) Trailing zeros or the zeros placed to the right of the number are significant.**

Example: 4.330 have four significant figures.

433.00 have five significant figures.

343.000 have six significant figures.

**(5) In expontential notation, the numerical portion gives the number of significant figures.**

Example: $1.32 \times 10^2$ have three significant figures.

$1.3234 \times 10^7$ have five significant figures.

- **Rounding Off:**

While rounding off measurements, we use the following rules by convention:

(1) If the digit to be dropped is less than 5, then the predceding digit is left unchanged.

Example:- x = 7.82 is rounded off to 7.8

again x = 3.94 is rounded off to 3.9

(2) If the digit to be dropped is more than 5, then the preceding digit is raised by one.

Example:- x = 6.87 is rounded off to 6.9

again x = 12.78 is rounded off to 12.8

(3) If the digit to be dropped is 5 followed by digits other than zero then the preceding digit is raised by one.

Example: x = 16.351 is rounded off to 16.4

again x 6.758 is rounded off to 6.8.

(4) If the digit to be dropped is 5 or 5 followed by zeros then preceding digit is left unchanged, if it is even.

Example: x = 3.250 becomes 3.2 on rounding off,

again x = 12.650 becomes 12.6 on rounding off.

(5) If digit to be deopped is 5 or 5 followed by zeros, then the preceding digit is raised by one, if it is odd.

Example:- x = 3.750 is rounded off to 3.8.

again x = 16.150 is rounded off to 16.2

- **Significant Figures in Calculation:**

In most of the experiments, the observations of various measurements are to be combined mathematically, i.e. added, subtracted, multiplied or divided to achieve the final result. Since all the observations in measurements do not have the same precision, it is natural that the final result cannot be more precise than the least precise measurement. The following two rules should be followed to obtain the proper number of significant figures in any calculation.

(1) The result of an addition or subtraction in the number having different precisions should be reported to the same number of decimal places as present in the number having the least number of decimal places. The rule is illustrated by the following example:

(i) 33.3 ← (has only one decimal place)

3.11

+0.313

36.723 ← (answer should be reported to one decimal place)

Answer = 36.7

(ii) 3.1421

0.241

+ 0.09 ← (has 2 decimal places)

$\underline{3.4713}$ ← (answer should be reported to 2 decimal place)

Answer = 3.47

(2) The answer to a multiplication or division is rounded off to the same number of significant figures as possessed by the least precise term used in the calculation. The rule is illustrated by the following example:

(i) 142.06

$\underline{\text{x } 0.23}$ ← (two significant figures)

$\underline{32.6738}$ ← (answer should have two significant figures)

Answer = 33

(ii) 51.028

$\underline{\text{x } 1.31}$ ← (three significant figures)

$\underline{66.84668}$

Answer = 66.8

(iii) $\frac{0.90}{4.26} = 0.2112676$

Answer = 0.21

- **Order of Magnitude:**

In scientific notation the numbers are expressed as, Number = M x $10^x$. Where M is a number which lies between 1 and 10 and x is integer. Order of magnitude of quantity is the power of 10 required to represent the quantity. For determining this power, the value of the quantity has to be rounded off. While rounding off, we ignore the last digit which is less than 5. If the last digit is 5 or more than five, the preceding digit is increased by one. For example.

(1) Speed of light in vacuum

= 3 x $10^8$ $ms^{-1}$ = $10^8$ m/s (ignoring 3 < 5)

(2) Mass of electron = 9.1 x $10^{-31}$kg = $10^{-30}$ kg (as 9.1 > 5)

- **Errors of Measurement:**

The measuring process is essentially a process of comparison. Inspite of our best efforts, the measured value of a quantity is always somewhat different from its actual value or true value. This difference in the true value and measured value of a quantity is called error of measurement.

## Types of Errors:

The errors in the measurement can be broadly classified as:

1. Systematic errors
2. Random errors
3. Gross errors

**1. Systematic errors** : The systematic errors are those errors that tend to be in one direction, either positive or negative. Infact, the causes of systematic errors are known. Therefore, such error can be minimised. Some of the sources of the systematic errors are:

**(a) Instrumental error**: which arise from the errors due to imperfect design or manufacture or calibration of the measuring instrument for example an ordinary metre scale may be worn off at one end in a vernier callipers the zero mark of vernier scale may not conincide with the zero mark of the main scale; the temperature graduations of a thermometer may not be accurately calibrated and so on. The instrumental error can be reduced by using more accurate instruments and applying zero correction etc when required.

**(b) Personal errors** arise due to inexperience of the observer for defect in the eyes of observer. For example lack of proper setting of apparatus reading and instrument without seeing it properly taking observation without observing proper precautions etc.

**(c) Errors due to external causes**: the external conditions such as changes in temperature, pressure, humidity, wind velocity etc. during the experiment may affect the measurements

**(d) Fixed error**: If the same error is repeated in all observations then it is called fixed error.

**2. Random Errors**: The random errors are the error which occurs irregularly due to unpredictable variations in the experimental conditions. The random errors are sometime called the chance error. For example when the same person repeats the same observation, he may get different readings every time. Hence, the random error can be minimised by repeating the observation a large number of time and taking the arithmetic mean of all the observations.

Let a physical quantity be measured n times. Let the measured value be $a_1$, $a_2$, $a_3$,.. $a_n$.

The arithmetic mean of these of value is $a_m = \frac{a_1 + a_2 + \ldots a_n}{n}$

3. **Gross error:** This error arises on account of share carelessness of the observer. For example:

- Reading and instrument without setting it properly.
- Taking the observation wrongly without caring for the sources of errors and the precautions.
- Recording the observations wrongly.
- Using wrong values of the observation in the calculations.

**ABSOLUTE ERROR, MEAN ABSOLUTE ERROR, RELATIVE ERROR AND PERCENTAGE ERROR**

(1) **Absolute error:** Absolute error in the measurement of a physical quantity is the magnitude of the difference between the true value and the measured value of the quantity.

Let a physical quantity be measured n times. Let the measured value be $a_1$, $a_2$, $a_3$,.. $a_n$. The arithmetic mean of these of value is $a_m = \frac{a_1 + a_2 + \ldots a_n}{n}$

Usually, $a_m$ is taken as the true value of the quantity, if the same is unknown otherwise.

By definition, absolute errors in the measured valued of the quantity are.

$\Delta a_1 = a_m - a_1$

$\Delta a_2 = a_m - a_2$

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$\Delta a_n = a_m - a_n$

The absolute errors may be positive in certain case and negative in certain other cases.

(2) **Mean absolute error**: It is the arithmetic mean of the magnitudes of absolute errors in all the measurements of the quantity. It is represented by Δa. Thus,

$$\overline{\Delta a} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots . |\Delta a_n|}{n}$$

(3) **Relative error or Fractional error:** The relative error or fractional error of measurement is defined as the ratio of mean absolute error to the mean value of the quantity measured. Thus,

$$\text{Relative error or Fractional error} = \frac{\text{Mean absolute error}}{\text{Mean value}} = \frac{\overline{\Delta a}}{a_m}$$

(4) **Percentage error:** When the relative/ fractional error is expressed in percentage, we call it percentage error. Thus

Percentage error = $\frac{\Delta a}{a_m}$ x 100%

- **Propagation of Errors:**

(1) **Error in sum of the quantities**: Suppose x = a +b

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

i.e. sum of a and b

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of x = $\frac{(\Delta a+\Delta b)}{a+b}$ x 100%

(2) **Error in difference of the quantities:** Suppose x = a-b

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

i.e. difference of a and b

The maximum absolute error in x is $\Delta x = \pm(\Delta a + \Delta b)$

Percentage error in the value of x = $\frac{(\Delta a+\Delta b)}{a-b}$ x 100%

**(3) Error in product of quantities:**

Suppose x = a x b

Let Δa = absolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

i.e. product of a and b

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm \left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x

= (% error in value of a) + (%error in value of b)

**(4) Error in division of quantities:**

Supose x= $\frac{a}{b}$

Let Δa = asolute error in measurement of a

Δb = absolute error in measurement of b

Δx = absolute error in calculation of x

i.e. division of a and b

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(\frac{\Delta a}{a} + \frac{\Delta b}{b}\right)$

Percentage error in the value of x

= (% error in value of a) + (% error in value of b)

**(5) Error in quantity raised to some power:**

Suppose x = $\frac{a^n}{b^m}$

Let $\Delta a$ = asolute error in measurement of a

$\Delta b$ = absolute error in measurement of b

$\Delta x$ = absolute error in calculation of x

The maximum fractional error in x is $\frac{\Delta x}{x} = \pm\left(n\frac{\Delta a}{a} + m\frac{\Delta b}{b}\right)$

Percentage error in the value of x

= n (% error in value of a) + m (% error in value of b)

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